Modular Robot Connector Area of Acceptance from Configuration Space Obstacles

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***Abstract*— Physical connectors which are robust to errors in position have the potential to contribute to the capabilities of many robot designs. This is especially true for modular robots, which frequently make and break connections. Determining the precise robustness of these connectors to errors has been a challenging topic for researchers, with many limited to categorizing the error tolerance in terms of single dimensions. We show a method for determining very closely the region of error tolerance (area of acceptance) in multiple dimensions by construction of a configuration-space model for a connector pair. Watershed analysis is then run on this configuration-space model to determine the full shape and area of the region of capture for the connector pair. We show the results of this process for several connector types.**

I. Introduction

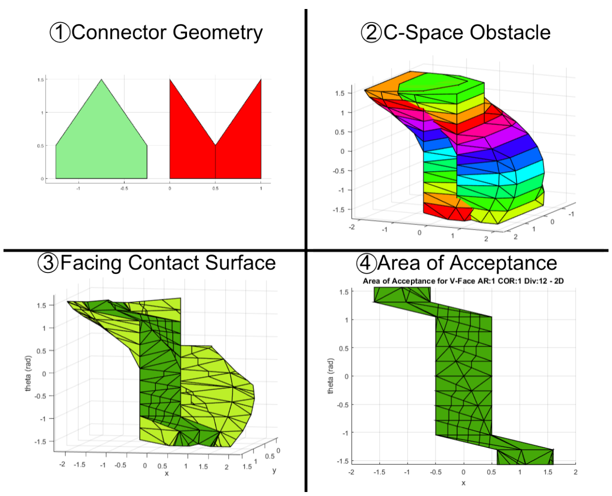

Fig. 1: Four of the steps to finding the Area of Acceptance

II. Related Work

The particular approach and application presented in this paper are unique, but there exists a significant history of work for both configuration-space representations and watershed algorithms in robotics literature.

Configuration-space methods are common in robotics applications, often used to simplify collision checking in path planning or inform kinematic reasoning for a system. Accurate computation of this configuration space in these cases is important, but potentially computationally intensive. Techniques to quickly compute the bitmap representation of the configuration space have been developed using the FFT [1] and polynomial transforms [2]. Polygonal boundary representation is somewhat more common - it is more accurate at the small scale, and allows for analysis of contact mechanics based on the shapes of the space formed. In particular, each surface represents a specific contact condition. It is also more useful in situations where the space and robot do not lend themselves to discretization. An early but detailed survey of configuration space methods can be found in [3]. Of note from this survey is that out of the 27 papers surveyed, only 1 [4] successfully represented the fully-dimensioned(that is, 6D) C-space for a 3D polygonal robot.

Watershed algorithms are used in computer vision and image processing applications, in particular morphological segmentation.

Similar techniques based on stability of objects in contact have been performed in other contexts. Kreigman [5] uses a capture region approach with an assumption of dissipative dynamics to calculate the stable positions of shapes against a plane along with their capture regions; however his approach is difficult to adapt to two arbitrary shapes. Stability tests based on the configuration space are well known. Mason, Rimon, and Burdick [6] [7] use configuration space representations to form a first and second order mobility theory suited for robotic grasps. In a separate work from the same authors [8], planar objects are examined under a potential field for stability. Unfortunately, these tests determine the stability rather than the capture region of the stable configurations.

Computational geometric toolboxes were necessary in order to complete our analysis. In particular we made use of the Multi-Parametric Toolbox for our computations, which contains functionality for representation, intersection, and Minkowski sum operations on these sets of polyhedra [9]. The robust and optimized implementation of these operations proved essential to the timely completion of the analysis.

III. Background

The docking problem is the process of creating the intended physical connection between two modules or robots. The problem can be broken down into two separate parts: *alignment* (or *mating*) and *attachment*. The first part means bringing the connectors into close enough proximity for the second part, where the physical connection is made. **Area of Acceptance** (AA) has been defined in previous work: 'Given some approach condition and pair of docking objects, the AA is the set of initial poses (relative to each other) that result in intimate alignment of the two parts.' [10]. The approach

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condition is the direction or path the two objects take towards each other.

In this work, we examine 2-dimensional rigid connector geometries. The connectors are controlled in the direction of approach with the other translation and rotation DOF free to move. These objects then come into contact and contact forces cause some relative motion until a stable state is reached, possibly the alignment state. We make some assumptions; namely that the motion is quasi-static and fully damped with no friction, coefficient of restitution, or dynamic/inertial effects. Since the geometric features of the connector effect the contact forces, we want to know if these features will cause the two objects to mate under given initial conditions. Initial conditions are defined as an offset in the relevant DOF (rotation and translation) from the perfect alignment state.

The **configuration space** is the state of all possible combinations of translation and rotation of a robot. For an obstacle in the workspace, we would like to determine to determine the states where the robot would be in contact with that obstacle. Therefore we form a *configuration space obstacle* by growing it according to the shape of the robot; the new shape is formed from the space taken up by the robot when it is overlaid on top of each point of the obstacle.

IV. Method Description

Analyzing the area of acceptance involves the two connecting objects in contact. The configuration space methods develop a model of two objects in contact. Contact analysis can then be done on this one surface of the c-obstacle which has all poses and surface interactions between the two objects encoded. This is particularly useful when considering combined rotations and translations of the objects in which docking behavior is unintuitive.

When two objects are in contact, docking along the controlled direction can be interpreted as motions along the C-obstacle surface in which the dot product of the surface normals with the docking direction vector is negative. If the target docked configuration is at a local minima (which is the desired design criteria) then all connected areas that have the negative dot product condition will lead to that docked configuration. In a height field this would be termed a watershed. Once this watershed is found on the surface, we define the area of acceptance as the projection of this watershed onto a plane perpendicular to the docking direction.

A. Key Concepts

Our method utilizes the idea of configuration space to analyze a pair of connectors. One connector is considered to be non-moving or the 'obstacle', and the other is considered to be moving, or the 'robot'. We can apply this to situations in which both connectors can move without loss of generality, by choosing the appropriate frame of reference.

We make additional assumptions about our connectors in the 2D case order to facilitate our analysis:

- The center of rotation of the connector moves strictly downward (in y) during the alignment phase. Note that this is not necessarily the center of the connector, which may in fact go up as the center of rotation is forced down.
- The center of rotation of the connector moves downward in a straight line until contact between the connectors occurs, at which point interaction forces allow it to move in x. In other words; there is a single straight-line *direction of approach* in the negative y direction.

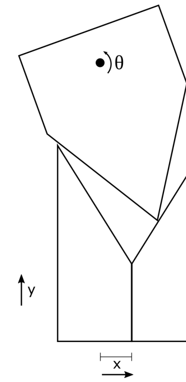

Fig. 2: Contact state visualization of a connector pair and offset coordinate system. Bottom connector shown here split into the union of two convex pieces for use within computational toolboxes.

The domains we seek to determine area of acceptance for are offsets from the single straight-line direction of approach for mating in the relevant domains. In the 2D connector case considered in this paper, the approach direction is y, with offsets possible in the x and θ directions. The centers of rotation advance directly towards one another in the approach direction with offset values x and θ held constant until contact is made. Motion continues monotonically along the direction of approach, free to move in the error domains. Eventually the two connectors reach a stable configuration or fall away from each other completely. Stable configurations are represented in the configuration space by local minima, and we call the desired stable configuration the 'mating configuration'.

There are two relative frames of reference for discussing the area of acceptance - the 'center of rotation frame' and the 'center of face frame' Depictions of the two frames are shown in Figures 3 and 4. You can see how the different frames affect the configuration space representation in **??**. Arms are considered 'out of plane' and do not interact - collisions are only possible between connector surfaces.

Each pair of connectors has two design parameters. *Aspect ratio of features* is defined as $\frac{H}{D}$, where H is vertical difference between maximum and minimum points on the face, and D is width of the connector. *Center of rotation distance* is defined as distance from the center of the connector to center of rotation, normalized by the height of the connector. Center of rotation points 'behind' the face of the connector are given negative values, where as center of rotation points 'in front' of the connector are given positive values.

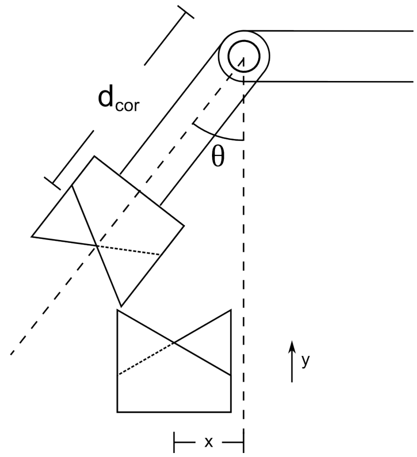

Fig. 3: Example of the 'center of rotation frame'. Note that x is measured between center of rotation of the moving piece and the center of the fixed piece.

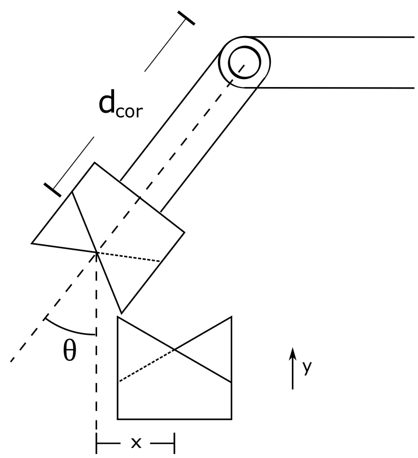

Fig. 4: Example of the 'center of face frame'. Note that x is measured between center of the moving piece face and the center of the fixed piece face.

B. Method Steps

This method has six steps:

- Description of parts and center of rotation
- Generation of C-Space representation
- Conversion to boundary
- Reduction to area of interest
- Prepartitioning of polyhedra
- Watershed determination by Two-Rule Algorithm
- Projection of watershed to error domain

The first step is purely definitional: here we define the geometry of the connectors in a concrete way along with their center of rotation.

V. Generation of Configuration Space Representation

The configuration space representation is generated from the description of the connectors. First we generate a fully rotated representation of the 'moving' connector. The *fully rotated representation* is the geometry of the connector extended in extra dimensions according to the directions in which rotation is possible (Fig. 5). The rotation is bounded to regions where mating might be possible, i.e. $\frac{-\pi}{2}$ to $\frac{\pi}{2}$. Since continuous representation of the rotation is impossible, we must carefully choose a resolution to approximate the rotation.

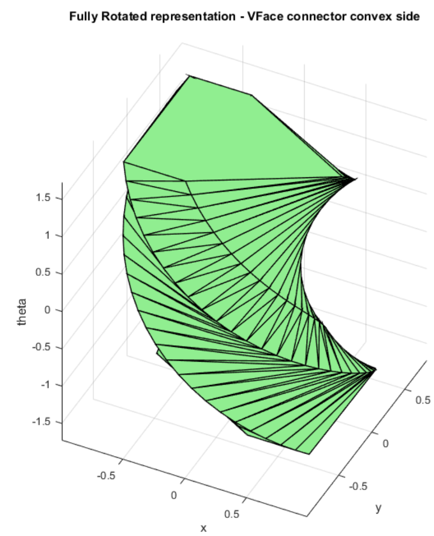

Fig. 5: Example of a rotated representation

Now that we have a fully-dimensional representation of one connector, we perform a Minkowski sum of the fully rotated representation and the lower-dimensional representation of the non-moving connector. If multiple convex polyhedra make up one or both of the connectors, a union operation is also performed involving these polyhedra.

Now we have a fully-dimensional configuration space representation of the connectors. Looking at the example in Figure 6, we can see there is a concavity surrounding the mating configuration. This concavity represents part of the *watershed* for the mating configuration - the set of points which will 'flow' down to it.

We have used this method to generate multiple configuration space obstacles for pairs of 2D connectors as well as 3D connectors. To the authors' knowledge, the *full* (6dof) configuration space obstacle for 3d shapes has not been generated since 1985 [4].

We now can reduce the C-space obstacle to its outer boundary. The boundary is found by intersection of the obstacle and the closure of it's complement. If we know our mating configuration in advance (which we often do), we can remove all parts of the boundary below that, which by definition cannot be in the watershed.

VI. Watershed Algorithm

A. Pre-partitioning

Now that we have the configuration space representation in a manageable form, we want to determine the watershed for

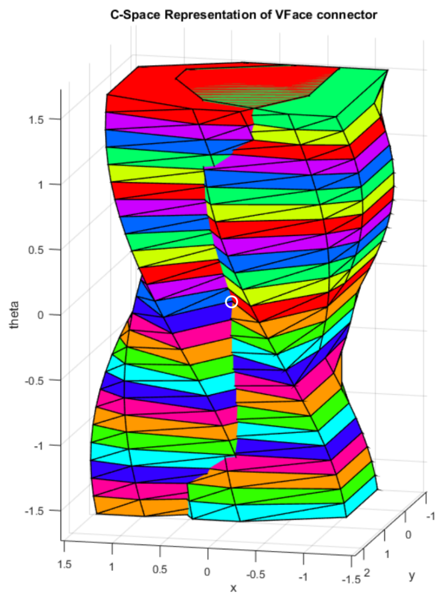

Fig. 6: Configuration space representation. Mating configuration indicated by a white circle.

our mating configuration. Historically, watersheds on data sets are found by use of the existing points with flooding algorithms. Our representation is different from typical sets of vertices analyzed for watersheds in two ways. Our vertices are set in arbitrary convex polyhedra rather than on a grid, and there exists a likelihood the representation is relatively sparse. This can lead to some incorrect watershed assignment if traditional flooding algorithms are applied without the appropriate rules for construction of dams. See Figure **??** for an example.

We have created and implemented here a new method for determination of the watershed on a set of convex polyhedra. The first part of this determination consists of pre-partitioning the set of polyhedra on the boundary such that each polyhedron has no more than one 'downhill' polyhedron. Each polyhedron by definition has a direction of steepest descent, and thus a (possibly empty) set of neighboring polyhedra to which points strictly descending along the polyhedra in this direction will go. The prepartitioning step consists of splitting polyhedra until each polyhedron has no more than one downhill polyhedron for all points inside it. Any point in this polyhedron then would travel to the same resultant polyhedron along its path of motion. Once partitioned, we can analyze each polyhedron as a whole rather than having to look at points independently

The prepartitioning allows us to say that each polyhedron has one and only one 'next' polyhedron on its path.

B. Graph Traversal Algorithm for Watersheds

With each polyhedron leading to no more than one other, the set now resembles a *directed pseudoforest* graph structure. Directed pseudoforests are defined as graphs in which each vertex has no more than one outgoing edge. We can perform a search on this graph structure to find a watershed, with a few extra rules specific to the geometric conditions.

In particular, these rules are based on a geometric condition which requires us to alter the graph structure slightly before assigning watersheds:

1) Ravine Condition: When two polyhedra point to one another in this set, they have some line connecting them that must be 'downstream' of both. The points on these polyhedra then would normally flow down along this 'ravine' line until they reach another polyhedron. So in the case where two polyhedra point to one another, we must go back to the geometry and find the new polyhedron they then point to. We call this a **ravine condition**. In order to determine the result of the ravine condition, we simply find the lowest point on the 'ravine' line r_{-}. If this point is contained in one other polyhedron, the two polyhedra then have their corresponding outward edges reassigned to this new polyhedron.

It is however possible, even likely, that this point will be contained in more than one polyhedron. If this is the case, we must choose some polyhedron to proceed. So we choose the polyhedron with the steepest downhill slope, as it represents the most likely direction for the motion to proceed in should some instability be introduced (as it often is in real cases).

2) Base Watersheds: After applying the ravine conditions, certain geometric cases present **base watersheds** by having no further polyhedra to which they will flow. There are two cases in which we can declare a base watershed reached. The first case is one in which the polyhedron points to nothing, or equivalently the vertex on the graph has no outgoing edge. This case most often occurs on the outside of the geometry, which is equivalent to the motion of the two connectors falling away from each other. Polyhedra which satisfy this condition are assigned to a single 'outside' watershed. The second occurs when polyhedra point to each other in a loop containing more than two polyhedra. In this case we have reached a set around a single watershed point, and it is simply the case that these polyhedra are sides of a single minima.

VII. Testing and Results

A. Relevant Geometry

In the interest of examining simple and representative examples of connector geometry, we selected for analysis two connector shapes - the V-Face and X-Face. These connectors were analyzed using a more dynamic method in a previous paper [11]. They are chosen for having simple geometry and a proven maximum possible lateral offset allowing successful self-alignment. Representations of the geometries and their lateral offsets are shown in Figure 7. Note that the X-Face is actually a 2D connector with two layers, giving it a wider allowable lateral offset. In practice, this means the X-Face requires some special handling - we generate the contact space for each layer separately and then take the union. The intended use of this method is to compare different connector shapes (ie. X-Face, V-Face) as well as compare the same shapes across two important design parameters - *aspect ratio of features* and *center of rotation distance*. These parameters are defined below.

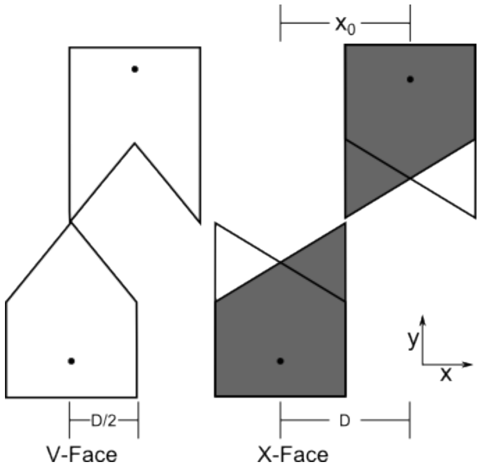

Fig. 7: Two-dimensional connector geometries that were tested.

B. Results

Results of the procedure were found for the V-Face and X-Face. These results are quantified as an area value in Tables I and II, with the plots showing the final shapes in Figures 8. Comparing to the results from the dynamic simulations performed in [11], we can see that most of the areas of acceptance are larger by a factor of up to two, as expected. Counterintuitively, four of the values in the lower left section of the table are larger for V-Face rather than X-Face.

After close examination of the C-space obstacle we conclude this is due to the change from dynamic to pseudostatic analysis. Several large patches reach critical points which in the dynamic case would pass into the area of acceptance, but in the pseudostatic case do not. The results show that this method is capable of determining the area of acceptance of geometrically defined connectors within a certain level of accuracy.

	COR:-1	COR:-1/2	COR:0	COR:1/2	COR:1
AR:1/4	0.38279	0.36586	0.85202	3.01069	3.14159
AR:1/2	0.40842	0.41129	0.86089	1.70012	2.77135
AR:1	0.43884	0.41643	0.87031	1.84022	3.14159
AR:2	0.45084	0.88724	0.84957	1.34941	3.14159
AR:4	0.46520	0.93465	0.88223	1.24182	3.14159

TABLE I: AA computed for V-Face

	COR:-1	COR:-1/2	COR:0	COR:1/2	COR:1
AR:1/4	1.09399	1.13635	1.28541	1.48352	1.84839
AR:1/2	0.97134	1.71965	1.99431	2.09141	2.27377
AR:1	0.92277	0.92203	1.79100	2.90494	3.13473
AR:2	0.88932	1.74696	1.66098	2.79853	4.09213
AR:4	0.89578	1.76955	1.65310	2.29481	4.65092

TABLE II: AA computed for X-Face

VIII. Conclusions

We introduce a new method of analysis for the area of acceptance of mechanical connectors under alignment. The method makes use of the configuration space obstacle representation of the connector pair and watershed determination methods to find the region of attraction also known

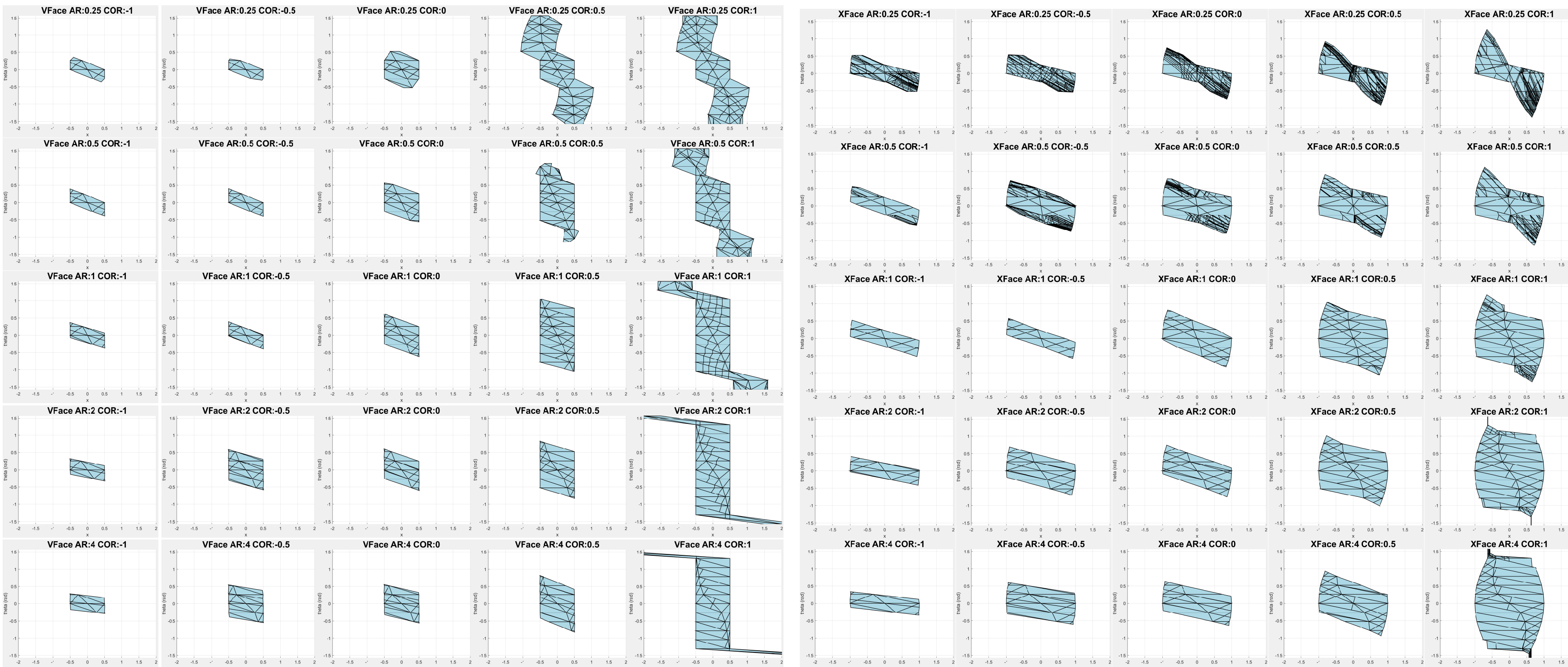

Fig. 8: Areas of acceptance for V-Face (left) and X-Face (right)

as the area of acceptance. The analysis is valid given several idealized assumptions and creates a representation of the configuration space within some resolution. From the configuration space obstacle representation we present a pre-partitioning method that divides the polyhedra making up the boundary such that each one has one 'downhill' polyhedra. We can then represent the set of polyhedra as a searchable graph with some modifications based on geometry. Search operations on the graph lead each polyhedra to its watershed assignment. We performed this method on two different connectors with a variety of design parameters. The result is a metric we can use to compare the connectors and their range of acceptance. This information allows us to make informed decisions about connector design parameters in robotic contexts.

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